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A Guided Search Framework in Multiple Model Control*

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Abstract— **The idea of using multiple models to cope with parameter uncertainties in adaptive systems was first introduced in the 1990s. Conventionally, methods based on this mechanism typically suffer from "curse of dimension," which means that the number of required identification models grows exponentially with respect to the number of unknown parameters. In this paper, the parameter identification problem is formulated as a time-varying optimization procedure, and a guided multiple model search framework is proposed to solve it. Instead of sampling the identification models in a large parameter space, models are sampled locally and used to estimate the search direction. As a result, the number of needed identification models grows linearly in this approach, in comparison with the exponential growth of existing methods. The proposed method also provides a unified form for nonlinear systems with non-affine unknown parameters, which is out of the scope of classical adaptive control theory. Moreover, theoretical convergence analysis is provided with specific conditions. The effectiveness of the proposed approach is verified by simulations and comparisons with existing methods.**

I. INTRODUCTION

Coping with systems with parameter uncertainties is a long-standing problem in control theory and applications. For systems with affine unknown parameters, it is well known that the classic adaptive control is an effective solution. When parameter initial errors are large or systems are oscillatory, multiple models (observers, agents) based control schemes [1-10], were introduced to improve the transient performance.

Multiple-model based control was first introduced by Athans et al. [1] on the design of Multiple Kalman Filters, where no switch among models was involved and no guarantee of stability was considered, similarly in [2], [3] and [4]. In the 1990s, Narendra and Balakrishnan [5] incorporated model switching and tuning into multiple model control framework, which aimed to accelerate the convergence rate of the parameter estimation. Following this, several variants were developed, including switching among fixed models ([6] and [7]), switching and tuning among adaptive models ([8] and [9]), and the hybrid combination of fixed and adaptive models. The common principle of these approaches is to sample the identification models in a large parameter space, and choose one model at each instant to determine the control input. Besides, the concept of multiple models with switching and tuning was also applied in identification problems, including parameter estimation [10] and combined state and parameter estimation [11].

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Although many simulations and experiments, e.g. [12], [20] and [21], have shown that multiple model control performs satisfactorily if sufficient amount of identification models are used, there still exist two main challenges. First, to assure that at least one of the identification models is sufficiently close to the real system, the number of required identification models grows exponentially with the number of unknown parameters. Second, the identification models are usually sampled in a large parameter space. Consequently, different models might be too far away from each other to be relevant, which makes them almost "useless" in locating the unknown real system parameters. Namely, only the model with the best performance will be chosen, and other models are severely under-utilized. A second level adaption presented in [13] achieved fast convergence with only a small number of models. However, this approach is limited to linear time-invariant (LTI) systems only.

To address these challenges, we formulate the parameter identification problem as a time-varying optimization procedure, and utilize a guided multiple model search framework to solve it. Instead of sampling models in a large parameter space, local sampling strategy is used to make identification models close to each other. Then, multiple models are used to estimate the search direction, and the identification models are updated by line search. Under conditions of persistency of excitation, local convexity, and local Lipschitz continuity, the proposed method guarantees the convergence of parameter estimation. The estimated parameters can then be used to determine the control input online.

Compared with existing multiple model control schemes, the proposed method significantly reduces the number of required models from exponential complexity to liner complexity, as well as the amount of computation. Moreover, it also provides a unified form for nonlinear systems with non-affine unknown parameters, which is out of the scope of the classical adaptive theory. In various real-world systems, like neural network control, robotics, and nonlinear models in finance, classes of problems are arising where efficient algorithms for non-affine parameters identification are required. The method presented in this paper provides an efficient solution for those problems.

This paper is organized as follows. In Section II, the mathematical notations and preliminaries are introduced. The proposed search framework is explained in Section III. In Section IV, the proposed framework is incorporated into parameter identification, with theoretical analysis on boundness and convergence. Simulations are presented in Section V. Section VI concludes the paper.

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II. NOTATIONS ADN PRELIMINARIES

A. Notations

Throughout this paper, the following notations are used. Let $\mathbb{R}=(-\infty,\infty)$, $\mathbb{R}_{\geq 0}=[0,\infty)$, $\mathbb{N}=\{0,1,2,...\}$, and $\mathbb{N}^{+}=\{1,2,3,...\}$ be the set of real numbers, non-negative real numbers, natural numbers and positive natural numbers. For vectors $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$, (x_1,x_2) denotes the vector $[x_1^T,x_2^T]^T \in \mathbb{R}^{n_1+n_2}$. For a vector $x \in \mathbb{R}^n$, $\|x\|_k$ denotes the k-norm of the vector x. Some special cases include the infinity norm $\|x\|_\infty=\max(|x_1|,|x_2|,...,|x_n|)$, and the Euclidean norm $\|x\|_2=\sqrt{x_1^2+x_2^2+...+x_n^2}$, where $x_1,x_2,...,x_n$ are the elements of x. By default, $\|x\|=\|x\|_2$. A continuous function α: $\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$ is called a class $\mathcal{K}$ function, if $\alpha(0)=0$ and α is strictly increasing. Additionally, if $\lim_{x\to\infty}\alpha(x)=\infty$, then α is called a class $\mathcal{K}_\infty$ function. The hyper-ball centered at ζ with a radius of r is denoted as $\mathcal{B}(\zeta,r) := \{x | r \geq \|x-\zeta\|\}$. The hyper-cube centered at ζ with a length of $2l$ is denoted as $\mathcal{H}(\zeta,l) := \{x | l \geq \|x-\zeta\|_\infty\}$. For any $\Delta > 0$, $\mathcal{M}_\Delta$ denotes the set of piecewise continuous functions from $\mathbb{R}_{\geq 0}$ to $\mathcal{H}(0,\Delta)$.

B. System, Identification Model and Monitoring Signal

Consider the following deterministic system

$$\dot{x}(t) = f(x(t),p^*,u(t)) \tag{1}$$
$$y(t) = h(x(t),p^*) \tag{2}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $y(t) \in \mathbb{R}^{n_y}$ is the output vector, $p^* \in \mathbb{R}^{n_p}$ is the unknown parameter vector, n_p is the number of unknown parameters, and $f(x,p,u)$ and $h(x,p)$ are known functions. The unknown parameter vector p^* is assumed to be constant and belong to a known compact set Θ. For any initial state x_0 and piecewise-continuous input $u(t)$, the system generates a unique solution that is defined for $t \geq 0$. If the state is measurable, let $h(x,p)=x$ and $y(t)=x(t)$. Otherwise, only $y(t)$ is measured and $x(t)$ needs to be recovered from $y(t)$. As this paper is focused on parameter estimation, it is assumed a robust control input $u(t)$ is known to prevent $x(t)$ from divergence.

A set of identification models is constructed, each is associated with a parameter estimate p. Thus, the dynamic equation that governs all of the identification models is

$$\dot{\hat{x}}(t) = \hat{f}(\hat{x}(t),p,u(t),y(t)) \tag{3}$$

where $\hat{x}(t)$ is the state vector of the identification model, p is the corresponding parameter estimate, and $u(t)$ and $y(t)$ are the control input and the measured output of real plant (1). Note that the classic adaptive law might be inapplicable for such system as (1), since the unknown parameters p in $f(x,p,u)$ might be non-affine.

Let $\tilde{x}(t)=\hat{x}(t)-x(t)$ denote the state error, and $\tilde{y}(t)=\hat{y}(t)-y(t)$ denote the output error. The error dynamic is

$$\dot{\tilde{x}}(t) = \hat{f}-f = F(\tilde{x}(t),x(t),p,p^*,u(t)). \tag{4}$$

A basic assumption on the design of identification models is that if $\tilde{x}(t_0)=0$ and $p=p^*$ at time t_0, it must be $F(\tilde{x},x,p,p^*,u)=0$ and $\tilde{x}(t)=0$ for any $t > t_0$, i.e., zero response for correct parameters. Some typical designs of the identification models can be found in [6], [11] and [18].

To evaluate the performance of an identification model over a period of time, a monitoring signal is defined. Similar to [9], we adopt the following weighted norm error for each identification model as the monitoring signal

$$\mu(t_{\text{start}},t,p,\tilde{x}_{t_{\text{start}}}) = \int_{t_{\text{start}}}^{t} w(\tau)\|\tilde{y}(\tau)\|^2 d\tau \tag{5}$$

where t_{start} and t specify the time period, p is the parameter estimate associated with the identification model, $\tilde{x}_{t_{\text{start}}}$ is the initial state error, $w(\tau)$ is a bounded strict positive weight function. A widely used weight function is the exponentially decaying weight, namely

$$w(\tau) = \exp(-\lambda(t-\tau)) \tag{6}$$

where $\lambda > 0$, and the monitoring signal can then be computed as

$$\begin{aligned} \dot{\mu} &= -\lambda\mu + \|\tilde{y}(t)\|^2 \\ \mu(t_{\text{start}}) &= 0 \end{aligned} \tag{7}$$

If all the identification models are initialized identically at t_{start}, the explicit dependency of $\mu(t_{\text{start}},t,p,\tilde{x}_{t_{\text{start}}})$ over $\tilde{x}_{t_{\text{start}}}$ can be dropped. The motivation is that, when the identification models are initialized or redistributed, their initial states are set to $\hat{x}_0$, which is our best state estimate at time t_{start}. If the state is measurable, their initial states can be set to true state measured from the real plant. Thus, the monitoring signal can be written into $\mu(t_{\text{start}},t,p)$. When the context is clear, the monitoring signal is abbreviated into μ.

In multiple model control schemes, such as [8] and [9], the designer can use an arbitrary number of models to identify the plant, but only one controller to control it. It therefore follows that N identification models (with the same structure as defined in (3)) can be set up to provide N parameter estimates $p_1,p_2,...,p_N$. Based on the monitoring signal μ, one of the models is selected to determine the overall parameter estimate at any time instant t, and it is used at that instant to determine the control input $u(t)$.

III. GUIDED MULTIPLE-MODEL SEARCH FRAMEWORK

The parameter identification problem in multiple model control can be cast into a time-varying optimization:

$$\underset{p}{\text{minimize}} : \mu(t_{\text{start}},t,p) \tag{8}$$

where μ is the monitoring signal in (5), t_{start} is the start time, t is current time instant, and the parameter estimate p is the decision variable. From this perspective, existing multiple model control schemes, such as [8] and [11], typically use exhaustive search to solve (8). In these approaches, identification models are sampled in a large parameter space, such as the known compact set Θ, and one identification model with minimal μ is selected as the solution of (8) at any time instant t.

In this paper, we propose to use guided search to solve (8). The guide information is obtained from known models with their performance indices. To be more specific, the associated monitoring signal μ of each identification model might indicate its parameter estimation quality, i.e., how far is p away from p^* in some metric. Thus, along the direction that minimizes $\mu(t_{\text{start}}, t, p)$ over p, it is likely to find new identification models with better estimation quality, although (8) is time-varying. Moreover, the direction that minimizes $\mu(t_{\text{start}}, t, p)$ over p can be approximated by existing models.

Such concept can be naturally incorporated into multiple model control, in which different models are sampled in the parameter space, and their monitoring signals are computed. As shown in Fig. 1, iterative update is incorporated into multiple model algorithm with the following steps:

1) Local sampling and simulation: Let i be the iteration number, given a parameter estimate p_{i-1} where the gradient computation is anticipated, local sampling centred at p_{i-1} is performed, as shown in Fig. 1 (b). After sampling, these models are simulated alongside the real system for a period T and their associated monitoring signals (5) are computed.

2) Search direction computation: Given a set of n models sampled in the vicinity of p_{i-1} and their associated monitoring signals $\mu_j, j = 1, 2..n$. This step computes the direction Δp_{i-1} that locally minimizes μ. A straightforward choice of search direction is the negative gradient of μ. Other more sophisticated approaches include momentum gradient, adaptive gradient, etc.

To estimate the gradient of μ over $p \in \mathbb{R}^{n_p}$, direct finite difference method requires n_p+1 identification models, where n_p is the number of unknown parameters. However, μ might be corrupted by disturbance or noise. Therefore, we use $2n_p \sim 3n_p$ models and RANSAC [16] method for gradient computation in our implementation.

3) Line search: As shown in Fig.1 (d), the line search is used to locate new prospective model along the computed search direction Δp_{i-1}, i.e.,

$$p_i = p_{i-1} + \alpha \Delta p_{i-1} \tag{9}$$

where α is the search step. Line search should ensure the updated model $p_{\text{updated}} = p_i$ is in the known compact set Θ, otherwise α will be discounted by a damping factor $\beta < 1$. Local sampling at p_i is also required, as shown in Fig. 1.

The algorithm above is summarized in Algorithm. 1 at the next page. The proposed framework preserves a set of models. We name it $ModelSet$, which is initialized by local sampling at one or more initial guesses. The algorithm performs an iterative update of $ModelSet$. In each iteration, all the models are simulated for a period T, and their associated monitoring signals μ are computed. Then, the model with best estimation performance p_{best} is used to perform the update step, which results in a new model p_{updated}. After that, local sampling at p_{updated} is performed and all the newly generated models are inserted into $ModelSet$.

As mentioned by Buchstaller and French [14], a $ModelSet$ should have better performance bound than any of its subsets. Furthermore, a larger $ModelSet$ is more robust to disturbance and noise. However, the size of $ModelSet$ is limited by computational resource. Thus, some inferior models must be discarded as a trade-off among performance and tractability.

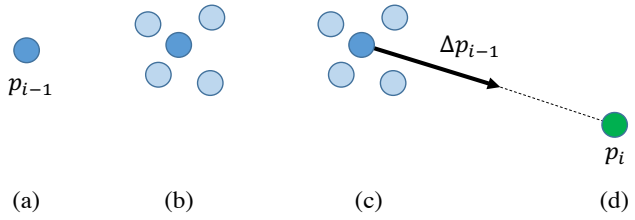


Fig. 1: Illustration of a sampling and update step. (a) The parameter estimate p_{i-1} (b) Local sampling and simulation (c) Search direction computation at p_{i-1} (d) Line search from p_{i-1} to p_i along Δp_{i-1}.

IV. PARAMETER IDENTIFICATION WITH GUIDED SEARCH FRAMEWORK

In this section, the proposed framework in Section III is incorporated into the parameter estimation problem. Boundness and convergence analysis of the parameter identification is also provided. For simplicity, our analysis is focused on the cases with minimal $ModelSet$, which means only one identification model and its local sampling models are preserved in $ModelSet$ at each instant.

The following assumptions are made concerning system (1) and the identification model dynamics (3). For simplicity, the monitoring signal (5) is abbreviated as $\mu(t, p)$ by setting $t_{\text{start}} = 0$, when the context is clear.

Assumption 1: The states of system (1) can be measured, thus $y(t) = x(t)$.

Assumption 2: For all $\Delta_x, \Delta_u \geq 0$, there exist a class $\mathcal{K}_\infty$ function $\alpha_{\tilde{x}}$ and a constant $T_f(\Delta_x, \Delta_u) > 0$, such that for all $p \in \Theta$, $x(0) \in \mathcal{H}(0, \Delta_x)$ and for some $u \in \mathcal{M}_{\Delta_u}$, the corresponding error $\tilde{x}$ satisfies: for any $t \geq T_f$

$$\int_{t-T_f}^{t} \|\tilde{x}(\tau)\|^2 d\tau \geq \alpha_{\tilde{x}}(\|p - p^*\|) \tag{10}$$

Remark 1: Assumption 2 is known as the *Persistency of Excitation* (PE) condition in adaptive literatures. It differs from classical PE in the sense that we consider a family of systems (4) parameterized by p. It is shown that assumption 2 can be verified by classic PE for some systems. One may refer Chong, et al [11] for more details.

The above assumptions ensure the unqiue optimality of the real system parameter p^*:

Proposition 1: If Assumptions 1 and 2 hold, there exists a time instant T_f, such that for any $t > T_f$, p^* is the unique minimizer of $\mu(t, p)$. Furthermore, for any $t > T_f$ and $p \neq p^*$,

$$\mu(t, p) \geq w \alpha_{\tilde{x}}(\|p - p^*\|) \tag{11}$$

where w is the minimum of weight function $w(\tau)$ over $[t - T_f, t]$. To show the convergence of parameter estimation

Algorithm 1: High Level Algorithm of Guided Search Framework

```
Initialize the ModelSet;
while system is running do
    Simulate all identification models for period T, compute monitoring signal μ;
    p_best ← Select_Best_Model(ModelSet);
    Δ_p ← Compute_Search_Direction(p_best);
    Check termination conditions;
    p_updated ← Linear_Search_Update_Model(p_best, Δ_p);
    ModelSet ← ModelSet ∪ Local_Sample_Models(p_updated, Θ, n);
    Discard some old models from ModelSet;
end
```

process, the following assumptions introduced in convex optimization [15] are adopted:

Assumption 3: Local Convexity. There exists a convex set $\Omega(t)$ in the vicinity of p^*, such that $p^* \in \Omega(t)$ and the function $\mu(t,p)$ is convex with respect to p in $\Omega(t)$.

Assumption 4: Continuity. For any t, the objective function $\mu(t,p)$ is differentiable with respect to p, and additionally

$$\|\nabla_p \mu(t,p_1) - \nabla_p \mu(t,p_2)\| \le L(t)\|p_1 - p_2\| \tag{12}$$

for any p_1, p_2 inside $\Omega(t)$, i.e., $\nabla\mu(t,p)$ is Lipschitz continuous with $L(t)$ at time t.

Remark 2: The continuity of $\mu(t,p)$ with respect to p at time t can be verified by the continuity and local Lipschitz continuity of function $F(\tilde{x},x,p,p^*,u)$ over p and $\tilde{x}$, where $F(\tilde{x},x,p,p^*,u)$ is defined in (4). The detailed analysis is presented in Appendix.

Remark 3: For nonlinear system with affine unknown parameters, $\mu(t,p)$ is a quadratic function of p:

$$\mu(t,p) = (p-p^*)^T Q(t)(p-p^*) \tag{13}$$

where $Q(t)$ is positive semidefinite for any $t \ge 0$. If $t > T_f$, where T_f is defined in assumption 2, then $Q(t)$ is positive definite. The convex set $\Omega(t)$ in assumption 3 is $\mathbb{R}^{n_x}$, $\nabla_p\mu(t,p_2)$ is a affine function $Q(t)(p-p^*)$ of p, and Lipschitz continuity of $\nabla_p\mu(t,p_2)$ holds. Thus, all above assumptions are satisfied. The analysis is shown in Appendix.

Based on the assumptions above, the convergence properties of the proposed search framework are discussed in the following subsections. Our discussion is divided into two cases, the offline identification and online identification, according to whether $u(t)$ and $x(t)$ are stored in the memory and can be used repeatedly.

A. Offline Identification

In offline identification, the input $u(t)$ and the state $x(t)$ of the original plant (1) are recorded from time t_{start} to t_{end}. Thus, the function $\mu(t_{\text{start}},t_{\text{end}},p)$ can be evaluated for any p by resetting $\hat{x}(t_{\text{start}}) = x(t_{\text{start}})$, simulating the system from t_{start} to t_{end} with parameter p, and computing the monitoring signal μ. As the recorded input $u(t)$ and state $x(t)$ from t_{start} to t_{end} are used, the dependency of Ω and L on t in (12) can be dropped, and both Ω and L are constants. Therefore, the parameter identification is formulated into an standard optimization problem, and its convergence can be stated as follows:

Proposition 2: In offline identification, with assumptions 1-4 satisfied, the parameter estimate p will converge to p^* if 1) p_0 is initialized inside Ω; 2) $\alpha \le 1/L$, where α is the search step in (9). The convergence rate satisfies

$$\mu(t_{\text{start}},t_{\text{end}},p_k) - \mu(t_{\text{start}},t_{\text{end}},p^*) \le \frac{\|p^0-p^*\|^2}{2\alpha k} \tag{14}$$

where k is the number of iterations. For the proof of this proposition, refer to Boyd and Vandenberghe [15].

B. Online Identification

In online identification, the search framework with multiple models is executed in parallel with the real system (1). $u(t)$ and $x(t)$ are obtained by measurement, and they are used to compute the monitoring signals μ of current identification models. No buffer or memory is required to store $u(t)$ or $x(t)$ in online identification.

In comparison with offline identification, online identification may be able to react to some time-varying effects, such as the slowly changing unknown parameters. Besides, online identification does not need any additional memory to store $x(t)$ and $u(t)$ for a long period with a high sampling rate. Thus, online identification is more suitable for real-time controllers.

Before presenting the main results of this section, some notations are clarified. Let t_0=0 be the initial time of the real system (1). At time t_i, where i=1, 2, ..., the search framework performs the ith update of the parameter estimate. Let p_i^- be the estimated parameter vector before update, and p_i^+ be the estimated parameter vector after update, hence $p_i^- = p_{i-1}^+$. The states of identification models are reset at t_i, thus $\tilde{x}(t_i^+) = 0$. The monitoring signals as functions of p are collected as $\mu_i(p)$=$\mu(t_{i-1},t_i,p)$, i=1, 2, ..., with associated Ω_i in assumption 3, weight function $w_i(t)$ in (5), L_i in assumption 4 and search step α_i in (9). Let $\Omega = \bigcap_{i=1}^{\infty}\Omega_i$. Ω is non-empty for $p^* \in \Omega$.

1) Boundness

Theorem 1: In online identification, with assumptions 1-4 satisfied, if 1) p^0 is initialized inside Ω; 2) $\alpha_i \le 1/L_i$, where α_i is the search step in (9), then

$$\|p_i^- - p^*\|^2 \ge \|p_i^+ - p^*\|^2 \tag{15}$$

in each update step i, and the inequality becomes equality if and only if $p_i^+ = p^*$.

Proof: See Appendix. □

2) Convergence

In the next theorem, the convergence of parameter estimate is established. The following assumption is made concerning the sequence $\{L_i\}$.

Assumption 5: The sequence $\{L_i\}$ is bounded by L. Thus, there exists a search step sequence $\{\alpha_i\}$ and a constant $c < 1/L$, such that $\alpha_i > c > 0, i = 1, 2, ...$

Remark 4: Assumption 5 means that it is possible to make a non-vanishing step for each parameter update. For nonlinear system with affine unknown parameters, assumption 5 can be ensured if the dwelling time $t_i - t_{i-1}$ is bounded, where $i = 1, 2, ...$ Refer to Appendix for detailed analysis.

Theorem 2: In online identification, with assumptions 1-5 satisfied, the parameter estimate p will converge to p^* if 1) p_0 is initialized inside Ω; 2) weight functions $w_i(t)$ are selected such that there exists a $0 < w \leq w_i(t)$ for any i and $t \in [t_{i-1}, t_i]$; 3) $c < \alpha_i \leq 1/L_i$, where α_i is the search step in (9).

Proof: See Appendix □

Remark 5: Unlike offline identification, online identification does not assure the boundness of the convergence rate.

Remark 6: Even if the parameter estimate does not converge, i.e. some assumptions such as the state measurability and the persistency of excitation are not satisfied, it can still be beneficial to use the proposed search framework. The idea is that the optimization procedure usually finds a better estimate in some metric compared with an initial guess, although it might oscillate in the vicinity of local optima.

V. SIMULATIONS

To illustrate the concepts discussed in the preceding sections, simulations are conducted on several examples. Online identification is used for all examples in this section.

A. A nonlinear system with affine unknown parameters

To test our convergence result presented in section IV, we tested on a nonlinear system with affine unknown parameters. The dynamics of the system is

$$\dot{x} = Ax + Bu + g(x, u)p^* \tag{16}$$

and the design of identification model is adopted from [18]

$$\dot{\hat{x}} = Ax + Bu + g(x, u)p + k(x(t) - \hat{x}) \tag{17}$$

where x=(x_1, x_2), B=$(0, 1)$, p=(p_1, p_2, p_3), $u \in$ R, and g_1=$(0, \sin(x_1 + x_2))$, g_2=$(0, \exp(x_1 + u))$, g_3=$(0, x_1^2 + x_2)$, k is a positive gain matrix. The value of true parameters are p_1^*=$12, p_2^*$=$2, p_3^*$=5. The initial guess is set as p=$(-5, -5, -7)$. A termination condition to stop the search procedure in our search framework is:

$$\|\nabla_p \mu\| < c_{\text{terminate}} \tag{18}$$

where $c_{\text{terminate}}$=0.3. The dwelling time is set to be 4s.

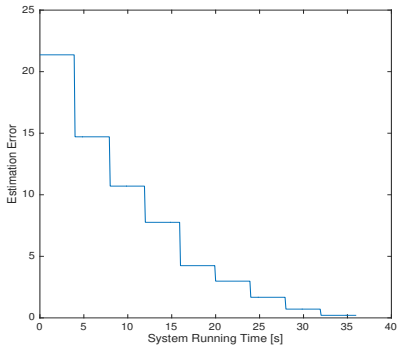


Fig. 2: Parameter estimation error of system in section V-A.

Figure 2 depicts the Euclidean norm of parameter estimation error $\|p - p^*\|$ over time. In the 9th iteration of search, the computed $\nabla_p \mu$ triggers the termination condition. The final estimated parameter is p=$(12.01, 1.93, 5.08)$.

B. A nonlinear disturbed system with non-affine parameters

To test the applicability and robustness of the proposed search framework, a simulation study was performed on the following nonlinear system with non-affine unknown parameters,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + p_1^*(x_1 + \sin(x_1)) + 14\sin(p_2^* x_2) \\ &\quad + \exp(p_3^* x_1) + \Delta \end{aligned} \tag{19}$$

with identification model dynamic

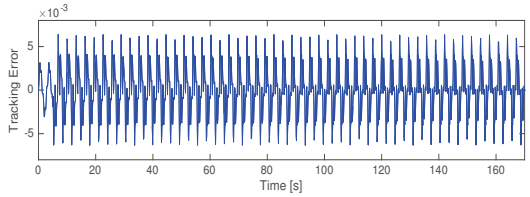
$$\begin{aligned} \dot{\hat{x}}_1 &= x_2 + k_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= u + p_1(x_1 + \sin(x_1)) + 14\sin(p_2 x_2) \\ &\quad + \exp(p_3 x_1) + k_2(x_2 - \hat{x}_2) \end{aligned} \tag{20}$$

where the true parameter vector is p^*=$(12, 2, 1)$, and Δ is a disturbance term. In this example, Δ is modeled as zero-mean Gaussian noise with standard deviation of σ=0.1. x_1 is driven to track a desired sinusoidal trajectory x_{des}=$\sin(2t)$. Totally nine identification models are used in parallel. The control input u is designed to be

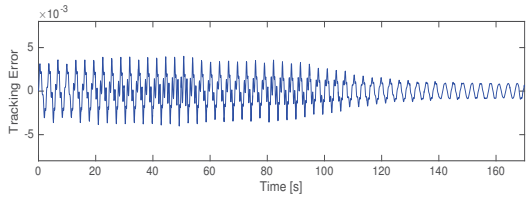
$$\begin{aligned} u &= u_{\text{feedforward}} + u_{\text{feedback}} + u_{\text{robust}} + u_{\text{compensate}} \\ u_{\text{feedforward}} &= \ddot{x}_{\text{des}} \\ u_{\text{feedback}} &= k_p(x_{\text{des}} - x_1) + k_d(\dot{x}_{\text{des}} - x_2) \\ u_{\text{compensate}} &= -(p_1(x_1 + \sin(x_1)) + 14\sin(p_2 x_2) \\ &\quad + \exp(p_3 x_1)) \end{aligned} \tag{21}$$

where $p = (p_1, p_2, p_3)$ is the parameter estimate, and the design of u_{robust} is described in [22]. The estimated parameter are initialized at p_0=$(8.5, 2.7, 3.0)$, the search step is α=0.03 and the dwelling time is T_f=4s.

The tracking error of the system with and without the proposed search framework is depicted in Fig. 3. The corresponding estimation error is depicted in Fig. 4. The final estimate is p=$(11.92, 2.12, 0.93)$. Due to the existence of noise, the parameter estimates do not converge to the true



(a) Tracking error of system in section V-B without the proposed search framework.



(b) Tracking error of system in section V-B with the proposed search framework.

Fig. 3: Tracking error of system in section V-B with/without the proposed search framework.

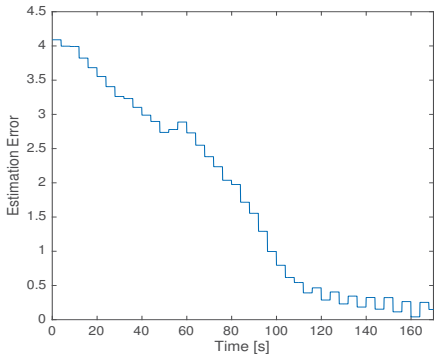


Fig. 4: Parameter estimation error of system in section V-B

values. However, we do obtain much better parameter estimates in comparison with the initial guess, and the transient performance of the controller is significantly improved.

C. Comparison with existing methods

As summarized in [14] and [17], model re-distribution methods were used in adaptive control to improve the transient performance. Ignoring the non-essential difference in their formulation, the central idea of these approaches is to move the identification models close to the one with the best identification performance.

To show the efficiency of the proposed method, a comparison is made between our method and the dynamic sampling approach in [11], which is a typical model-redistribution approach. The two algorithms are executed on the system described in section V-B. Our platform is a MacBook Pro with 2.7 GHz Intel Core Quad processor, running Matlab environment. The comparative result is shown in Table. I. From the comparison, the proposed method not only outperforms in parameter estimation accuracy but also significantly reduces computational load.

	Proposed	Dynamic Sampling
Number of used models	9	64 ($4 \times 4 \times 4$)
Computation time [s]	95	774
System running time [s]	170	170
Estimation error	0.16	2.03

TABLE I: Comparison between the proposed method and the dynamic sampling method on system (42). The proposed method provides better estimation accuracy, uses fewer identification models and saves computational resource in comparison with the dynamic sampling method.

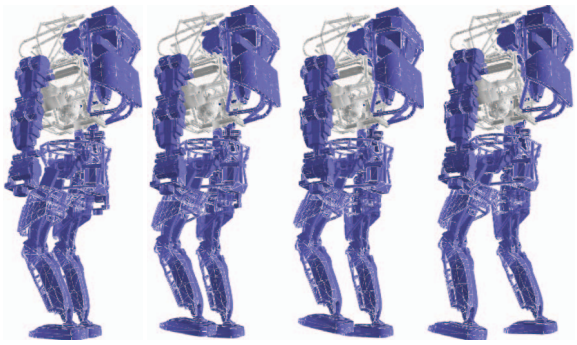


Fig. 5: The proposed method is applied to a simulated Atlas humanoid robot. Figure shows snapshots of a consecutive step of the walking task used as the exciting trajectory.

D. Application to a simulated humanoid robot

The proposed method is applied on the mass identification of several links of a simulated full-scale humanoid robot. The walking task shown in Fig. 5 is used as the exciting trajectory and the mass of seven links related to walking (torso, left/right foot, left/right calf and left/right thigh) are assumed unknown.

The dynamic equation of a humanoid is

$$H(q)\ddot{q} + C(q,\dot{q}) = \tau + J^T f \tag{22}$$

and the identification model is

$$\hat{H}(q)\ddot{\hat{q}} + \hat{C}(q,\dot{q}) = \tau + J^T f + K_1(q-\hat{q}) + K_2(\dot{q}-\dot{\hat{q}}) \tag{23}$$

where q and $\hat{q}$ are generalized positions, H and $\hat{H}$ are mass matrices, C and $\hat{C}$ are bias forces, τ is the torque input. f is the ground reaction force measured by simulated sensors and J is the contact Jacobian. The state vector is $x = (q, \dot{q})$. There are 7 unknown parameters and 24 models are used.

Figure 6 depicts the L2 mass error over time. The initial L2 mass error is approximately 6 kg. After more than 20 iterations, the L2 error is less than 0.5 kg.

VI. CONCLUSION

The contribution of this paper is two-fold: first, we introduce the guided search framework into multiple model control scheme. To our best knowledge of current literature, this is the first effort to perform guided search in multiple models based tuning and switching framework, and carry out the update of the model set by descent algorithm. Second, the

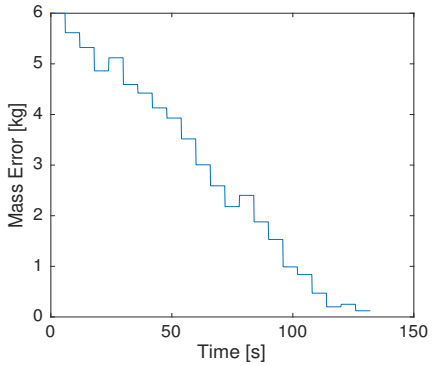


Fig. 6: Mass estimation error of the simulated humanoid robot.

convergence of parameter estimate under certain conditions is established. The number of needed identification models in the proposed method grows linearly with respect to the number of unknown parameters, in comparison with the exponential growth of existing methods. Thus, the proposed method shows significant improvement in computational efficiency. Besides, the proposed framework allows us to treat non-linear system with non-affine unknown parameters in a unified and modularized form, which circumvents the complicated and problem-specific linearization process. Simulation results demonstrate the efficiency and robustness of the proposed method.

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APPENDIX

A. Analysis of Remark 2

Remark 2 can be formally stated as follows. For any finite t, $\mu(t,p)$ is continuous with respect to p, if the function $F(\tilde{x},x,p,p^*,u)$ in (4) is continuous and locally Lipschitz continuous with respect to p and $\tilde{x}$, for the given state $x(t)$ and input $u(t)$ of the original plant (1).

Proof: Remark 2 will be proved by Poincare Map. Let

$$\tilde{x}(t_{\text{start}} : t_{\text{end}}, p, \eta) \tag{24}$$

denote the solution of (4) at time t_{end}, if (4) is initialized at $t{=}t_{\text{start}}$ with $\tilde{x}{=}\eta$ and parameter p, for given $u(t)$ and $x(t)$. By using Poincare Map theorem, $\tilde{x}(t_{\text{start}} : t_{\text{end}}, p, \eta)$ is continuous with respect to p and η.

As $[t_{\text{start}}, t_{\text{end}}]{\times}\Theta$ is compact, $\tilde{x}(t_{\text{start}} : \tau, p, \eta)$ is uniformly continuous with respect to τ and p. Thus, for any $p, \bar{p} \in \Theta$, $t_{\text{start}} \leq \tau \leq t_{\text{end}}$ and $\epsilon > 0$, there exists δ, such that $\|p - \bar{p}\| < \delta$ implies,

$$\|\tilde{x}(t_{\text{start}} : \tau, p, \eta) - \tilde{x}(t_{\text{start}} : \tau, \bar{p}, \eta)\| < \epsilon \tag{25}$$

Moreover, recall the definition of μ in (5) and let K be the upper bound of $w(t)$, for $t_{\text{start}} \leq \tau_i \leq t_{\text{end}}$. Then with (25), for any $p, \bar{p} \in \Theta$ and $\epsilon > 0$, there exist $\delta > 0$, such that if $\|p - \bar{p}\| < \delta$,

$$\|\mu(t_{\text{start}}, t_{\text{end}}, p) - \mu(t_{\text{start}}, t_{\text{end}}, \bar{p})\| < KT\epsilon^2 \tag{26}$$

where $T{=}t_{\text{end}}-t_{\text{start}}$. The continuity of μ over p is proved. □

B. Proof of Remark 3

Consider a non-linear system with affine unknown parameters, as studied in [18]

$$\dot{x}(t){=}f(x(t),u(t))+g(x(t),u(t))p^* \tag{27}$$

and the associated identification model

$$\dot{\hat{x}}(t){=}f(x(t),u(t))+g(x(t),u(t))p+k(t)\tilde{x}(t) \tag{28}$$

where $x(t)\in\mathbb{R}^{n_x}$, $u(t)\in\mathbb{R}^{n_u}$, $p\in\mathbb{R}^{n_p}$ are state vector, control input vector and unknown parameter vector, respectively. $f(x,u)\in\mathbb{R}^{n_x}$ and $g(x,u)\in\mathbb{R}^{n_x\times n_p}$ are continuous functions. $\tilde{x}(t){=}x(t)-\hat{x}(t)$ is the tracking error of identification model, $k(t)$ is a positive gain matrix in the form of $\text{diag}(k_1(t),k_2(t),...,k_{n_x}(t))$, $k_i(t)\geq 0$. The error dynamic is

$$\dot{\tilde{x}}(t)+k(t)\tilde{x}(t)=g(x(t),u(t))(p^*-p) \tag{29}$$

Suppose $x(t)$ and $u(t)$ are provided, (29) is a linear differential equation of $\tilde{x}$. $\tilde{x}(0){=}0$ because identification models will be reinitialized after each sampling. Thus, the solution of error dynamic (29) is linear with respect to $p{-}p^*$, which can be represented as

$$\tilde{x}(t)=G(t)(p^*-p) \tag{30}$$

and the monitoring signal is

$$\mu(t,p)=(p^*-p)^T\Big(\int_0^t w(\tau)G(\tau)^TG(\tau)d\tau\Big)(p^*-p) \tag{31}$$

The matrix $M(t){=}\int_0^t w(\tau)G(\tau)^TG(\tau)d\tau$ is positive semidefinite for $t\geq 0$. Thus, $\mu(t,p)$ is a quadratic function of p. If $t>T_f$, where T_f is defined in assumption 2, $\int_0^t w(\tau)G(\tau)^TG(\tau)d\tau$ is positive definite. For $t>T_f$ the problem has an unique solution $p{=}p^*$. □

C. Analysis of Remark 4

Consider the system studied in (27), from (31) one obtains

$$\mu_i(p)=(p^*-p)^T\Big(\int_{t_{i-1}}^{t_i} w(\tau)G(\tau)^TG(\tau)d\tau\Big)(p^*{-}p) \tag{32}$$

where $G(t)$ is defined in (30). Let $M_i=\int_{t_{i-1}}^{t_i} w(\tau)G(\tau)^TG(\tau)d\tau$, thus

$$L_i\leq\lambda_{max}(M_i) \tag{33}$$

where $\lambda_{max}(M_i)$ is the maximum eigenvalue of the positive semi-definite matrix M_i. If $x(t)$ and $u(t)$ are bounded, which can be achieved by robust control schemes like [22], $g(x(t),u(t))$ must be bounded, for the continuity of $g(x,u)$. Thus, $\tilde{x}(t)$ in (29) and $G(t)$ in (30) are bounded. If there exists a T_{upper}, such that $t_i-t_{i-1}<T_{\text{upper}}$, $i{=}1,2,...$, then M_i and all its eigenvalues are bounded. As a result, the sequence $\{L_i\}$ is bounded. □

D. Proof of Theorem 1

From assumption 4 and the equation 2.4 of [23], for any p_1,p_2 in Ω and $i\in\mathbb{N}_+$,

$$\begin{aligned}\mu_i(p_1)\leq&\mu_i(p_2)+\nabla\mu_i(p_2)(p_1-p_2)\\&+\frac{L_i}{2}\|p_1-p_2\|^2\end{aligned} \tag{34}$$

Let $p_1=p_2-\alpha_i\nabla\mu_i(p_2)$, thus for all p_1,p_2,

$$\mu_i(p_1)\leq\mu_i(p_2)-(1-\frac{L_i\alpha}{2})\alpha\|\nabla\mu_i(p_2)\|^2 \tag{35}$$

Take $p_1=p_i^+,p_2=p_i^-$, and letting the search step α_i satisfy $\alpha_i\leq 1/L_i$. Then,

$$\mu_i(p_i^+)\leq\mu_i(p_i^-)-\frac{\alpha_i}{2}\|\nabla\mu_i(p_i^-)\|^2 \tag{36}$$

By plugging in (34), and letting $p_2{=}p^*,p_1{=}p_i^-$, one obtains

$$\mu_i(p_i^+)\leq\mu_i(p^*)+\nabla\mu(p_i^-)(p_i^--p^*)-\frac{\alpha_i}{2}\|\nabla\mu_i(p_i^-)\|^2 \tag{37}$$

Notice that

$$\begin{aligned}\frac{1}{2\alpha_i}(\|p_i^--p^*\|^2-\|p_i^+-p^*\|^2)=&\nabla\mu_i(p_i^-)(p_i^--p^*)\\&-\frac{\alpha_i}{2}\|\nabla\mu_i(p_i^-)\|^2\end{aligned} \tag{38}$$

Inserting (38) into (37), one obtains

$$\mu_i(p_i^+)-\mu_i(p^*)\leq\frac{1}{2\alpha_i}(\|p_i^--p^*\|^2-\|p_i^+-p^*\|^2) \tag{39}$$

Thus, we reach the conclusion

$$0\leq(\|p_i^--p^*\|^2-\|p_i^+-p^*\|^2) \tag{40}$$

because $\mu_i(p_i^+)\geq\mu_i(p^*)$, and equality holds if and only if $p_i^+{=}p^*$. □

E. Proof of Theorem 2

Consider the sequence

$$a_i=\|p_i^+-p^*\|^2, i=0,1,2,... \tag{41}$$

As $p_i^-{=}p_{i-1}^+$, from lemma 1, if $a_{i+1}\neq 0$

$$a_{i+1}<a_i \tag{42}$$

By using $a_i\geq 0$, it is known that $\lim_{i\to\infty}a_i$ exists. Suppose $\lim_{i\to\infty}a_i{=}c>0$, and let another sequence $\{b_k\}$ be $b_k{=}(a_k-a_{k+1})/(2\alpha_i)$. By assumption 5, one obtains $\lim_{k\to\infty}b_k{=}0$. However, by plugging in (39), one obtains

$$b_k\geq\mu_i(p_i^+)-\mu_i(p^*)=\mu_i(p_i^+) \tag{43}$$

Thus, $\lim_{i\to\infty}\mu_i(p_i^+){=}0$. Suppose $\lim_{i\to\infty}(p_i^+)$ do not exist or $\lim_{i\to\infty}(p_i^+)\neq p^*$. Because the sequence $\{p_i^+\}$ is bounded, there must be a subsequence of $\{p_i^+\}$, namely $\{p_{i'}^+\}$, that $\lim_{i'\to\infty}(p_{i'}^+)$ exists and $\lim_{i'\to\infty}(p_{i'}^+)=p^{**}\neq p^*$. Since $\{p_{i'}^+\}$ is a subsequence of $\{p_i^+\}$, one obtains

$$\lim_{i'\to\infty}\mu_{i'}(p_{i'}^+)=0 \tag{44}$$

However, by Proposition 1, $\mu_{i'}(p^{**})\geq w\alpha_{\tilde{x}}(\|p-p^*\|)$, which contradicts (44). Thus, $\lim_{i\to\infty}(p_i^+){=}p^*$, and the proof is complete. □